Likelihood of winning and overbidding in first price auctions ☆

Paolo Crosetto, Antonio Filippin, Peter Katušcák

Economic theorists have suggested several decision-making models that can explain how bidders in first-price sealed-bid auctions (FPA) choose bids given their values. Many of such models posit a bidder objective that is linear in the probability of winning the auction conditional on own bid. The best-known of such models is expected utility (payoff) maximization. However, the same is true also of several other models, such as anticipated regret (EW 1989; Filiz-Ozbay and Ozbay 2007) and utility of winning.

The first goal of our experiment is to test this prediction. Such test is arguably difficult to conduct using data from a usual experimental implementation of FPAs that involves multiple human bidders competing for the object. This is because the tradeoff between bid and the probability of winning is usually clouded by several design features. First, bidders might be uncertain about their opponents' value distribution (Chen et al. 2007), even though in most of the experiments they are not. Second, even if bidders do know their opponents' value distribution, they are uncertain about bidding strategies used by the opponents. Third, even if they know the opponents' value distribution and their bidding strategies, they have to compute distribution of the maximum of several competing bids. Our design assumes away all such complexity by making each human bidder bid against *one* computerized opponent whose *bids* are drawn from a *known uniform distribution*. Moreover, such design rules out any impact of social preferences on bidding.

Since the probability of winning given own bid is central to our design, we go a step further by depicting this probability in a salient way to each bidder. In this interface, a bidder can try various potential bids. For each potential bid, he is given the payoff conditional on winning, the payoff conditional on losing and a graphical representation of the winning probability.

The main part of the design is based on two treatments. In treatment 60/60, each human bidder has a value of 60 whereas computer's bids are drawn from $U[0, 60]$. In treatment 60/100, each human bidder has a value of 60 whereas computer's bids are drawn from $U[0, 100]$. In both treatments, the human bidder can choose any bid from the set $\{0, 1, .., 100\}$. As a result, the only thing that is changing between the two treatments is the probability of winning and that of losing conditional on a given bid. The payoffs conditional on winning and losing remain the same. In particular, the probability of winning given a bid is 40 percent lower in 60/100 compared to 60/60.

☆ *Contact:* paolo.crosetto@grenoble.inra.fr (Paolo Crosetto), antonio.filippin@unimi.it (Antonio Filippin), peter.katuscak@cerge-ei.cz (Peter Katušcák)

May 31, 2014

All the models of bidding that posit an objective that is linear in the probability of winning given own bid predict that bidders should behave identically in the two treatments. Intuitively, in 60/100, one can simply condition bidding on computerized bid not exceeding 60 since in the case it does, there is no profitable way of winning anyhow. Contrary to this prediction, we find that bidders bid more in 60/100 (average bid of 35.2) than they do in 60/60 (average bid of 30.8). This result suggests such theories cannot (fully) account for behavior of bidders in auction experiments.

Given this finding, we consider theories that posit objectives that are non-linear in the probability of winning conditional on own bid. The first natural candidate is probability weighting (Kahneman and Tversky 1979). Using a two-parameter weighting function introduced by Lattimore at al. (1992), we find that probability weighting can theoretically explain our results. However, for the values of the two parameters commonly estimated in the literature, the predicted comparative static goes in the opposite direction to our empirical findings. Therefore, empirically, we do not consider probability weighting to provide an explanation for our finding. The second candidate is reference-dependence à la Koszegi and Rabin (2007). Using the parametrization introduced by Lange and Ratan (2010), we find that our finding is consistent with a very high degree of loss aversion, which although empirically plausible is not often encountered ($\lambda > 2.5$).

The second goal of our paper is to provide a rationale for the observed finding, given the null or weak support of the existing theories. We propose pessimism as a candidate to shape subjects' behavior. The intuition is that the perception of a lower likelihood of winning the auction extends to the whole domain of probabilities. A bidder tends therefore to underestimate his objective probability of winning the auction conditional on his bid. In our 60/100 treatment this implies that the subjects underestimate their probability of winning even when it should not be the case, i.e. when the computer bids less than 60. In contrast, such an effect does not operate when bidders feel that they have the same chances of winnning, like in the 60/60 treatment.

This mechanism represents a context-dependent distortion of probabilities. It other words the same probability can be perceived as both higher and lower than the objective one according to the relative strength of the subject in the auction. From a technical point of view it can be operationalized by a simple power function weighting of the probability of winning with the power increasing with the adversity of the bidding environment as represented by subjects' valuation v:

$$w(p) = p^{\frac{E(v_{-i})}{v_i}}$$

Besides allowing us to rationalize the bidding behavior in our experiment, such explanation can also account for several other findings observed in laboratory implementations of symmetric and asymmetric sealed-bid auctions with multiple human bidders.

This model finds prima facie supporting evidence from the perceived probability of winning the auction elicited at the end of our experiment. In fact, we find that the perceived probability of winning conditional on one's bid is significantly lower in the 60/100 treatment than in the 60/60 condition.

Take home slide

Objective:

- Explain overbidding in first price sealed-bid auctions with independent private values
- Use a laboratory experiment to rule out or to test traditional explanations

Preview of findings:

1. We deconstruct and simplify a first price auction to the point that we find negligible overbidding.
2. Overbidding emerges when the likelihood of winning is known but asymmetric
3. Predictions of traditional explanations are either directly rejected by our results or requiring improbable values of parameters.
4. We rationalize the results proposing a context-dependent probability distortion which depends on the symmetry of the bidding environment. Subjects overbid when pessimist about the likelihood of winning because they feel they are disadvantaged.

Motivation

Stylized fact:

In first-price sealed-bid auctions with independent private values, subjects bid more than what is predicted by the risk-neutral Bayesian-Nash equilibrium (Kagel 1995).

"Traditional" explanations:

- Risk aversion
  - Cox, Robertson and Smith (1982), Cox, Smith and Walker (1982)
- Incorrect perception of the bid/winning probability trade-off
  - Dorsey and Razzolini (2003), Armantier and Treich (2009)
- Anticipated loser regret
  - Isaac and Walker (1985), Ockenfels and Selten (2005),...
- Level-k thinking
  - Crawford and Iriberri (2007)
- Utility of winning/lack of outside options to make money
  - Cox, Smith and Walker (1988), Turocy and Watson (2012),...

Setup

Simple bidding environment:

- Computerized opponent that draws bids from a discretized uniform distribution
- Clearly communicated bid/winning probability trade-off

This setup rules out strategic interaction concerns and therefore:

1. Level-k thinking
2. Misperception of probabilities due to strategic interaction

Experimental treatments provide a direct test for the implications of

1. Expected utility theory
2. Anticipated loser regret
3. Utility of winning

Theoretical predictions of traditional explanations

Notation and assumptions:

- Human player's bid: $b \in [0, B]$
- Computerized opponent's bid: b_c drawn from a uniform $U[0, B]$
- Human player's value: v, with $v \leq B$ (crucial: it can be $v < B$)

Optimal bid under risk neutrality, no regret, and no utility of winning:

$$max_b (b/B)(v - b)$$

Optimal bid $b^* = v/2$

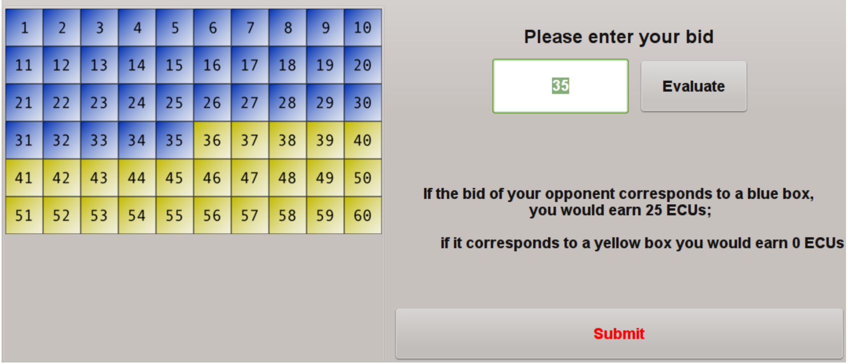
Optimal bid under risk aversion, anticipated regret, or utility of winning:

Optimal bid $b^* > v/2$

Testable implication:

The optimal bid-to-value ratio is a constant. In particular, it does not depend on the probability of winning conditional on one's bid b/B.
Rescaling such a probability leaves the optimal choice unchanged under these theories (and all the theories positing an objective function linear in the probability of winning).

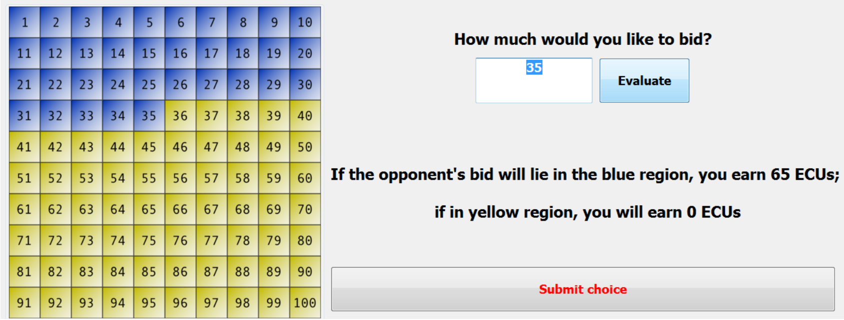
Symmetric Condition $v = B = 60$



Features:

- Evaluation of possible bids before submitting a payoff-relevant bid
- Straightforward representation of the payoff / winning probability tradeoff
  1. visual representation of the probability of winning
  2. explicit description of the payoff conditional on winning

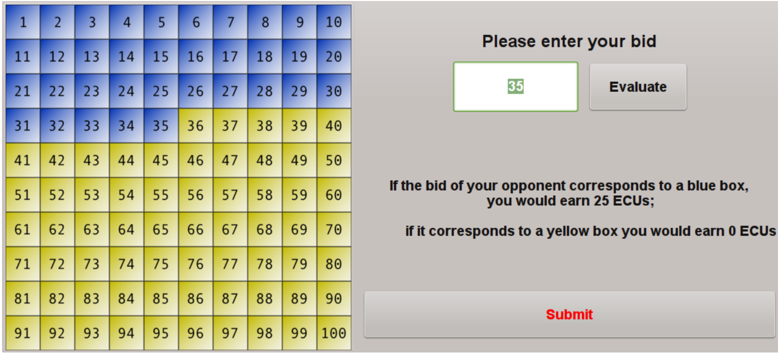
Symmetric Condition $v = B = 100$



Features:

- Evaluation of possible bids before submitting a payoff-relevant bid
- Straightforward representation of the payoff / winning probability tradeoff
  1. visual representation of the probability of winning
  2. explicit description of the payoff conditional on winning

Asymmetric Condition $v = 60 < B = 100$



Additional features:

- Loss warning in case of bidding more than the value

Additional Measures to Assure Subject Understanding

Control questions:

- 15 in total
- Payoff conditional on the own and opponent's bid
- Probability of the opponent's bid being in a certain range
- Independence of the opponent's bid from the own bid
- Tie-breaking
- Subjects not able to proceed until answering all 15 questions correctly
- Could consult an experimenter if stuck

Logistics

Subject pool:

- Between subject design
- 174 subjects (Friedrich Shiller University in Jena)

		v	
		60	100
B	60	55 subjects	
	100	59 subjects	60 subjects

Session outline:

1. Instructions and control questios (on-screen, in German)
2. Decision (one shot)
3. Questionnaire
4. Feedback
5. Payoff

Exchange rate:

- 1 ECU = 20 Euro cents

Testable implication

Testable implication:

According to the aforementioned theories subjects should always bid half of their value

Intuition

It is straightforward to show that both in the $60-60$ and in the $100-100$ risk neutral subjects should bid half of their value, i.e. 30 and 50, respectively.

When $v < B$, i.e. in the $60-100$ condition, subjects should realize that there is nothing they can do if the bid of the computerized opponent is greater than 60.

The conditional probability of winning is simply scaled down in the $60-100$ treatment as compared to the $60-60$ but the shape of the expected utility is the same, and therefore the choice should also be the same.

Results

Average choices

Treatment	N	bid-to-value Mean	StDev.
60-60	59	51.4	14.1
60-100	55	58.6	15.5
100-100	60	51.5	12.6

- Average choice very close to RNNE in the Symmetric treatments ($r \simeq .925$)
- Much stronger overbidding in the Asymmetric Condition
- The asymmetric treatment significantly differs as compared to both the $60 - 60$ (Mann-Whitney $p = 0.008$) and the $100 - 100$ ($p = 0.009$).
- The two symmetric treatments do not significantly differ ($p = 0.866$).

Testable implication is rejected

Results clearly reject the testable implication of the theories entailing linearity in the conditional probability of winning

Other theories: Probability weighting

Probability weighting fails empirically

- Probability weighting implies a non-linear relation with the conditional probability of winning and could therefore rationalize our results.
- However, the two-parameter functional form proposed by Lattimore (1992)

$$w(p) = \frac{\delta p^{\gamma}}{\delta p^{\gamma} + (1-p)^{\gamma}} \quad (1)$$

can rationalize a higher bid-to-value ration in the Asymmetric treatment only for parameter values that do not have empirical support (S-shape)

Other theories: Reference dependent preferences

Framework

This theory is relevant only to the extent that the bidder may perceive a loss as a result of not winning the auction.
We rely on a stochastic reference point given by the lottery that is actually induced by the chosen bid (Lange and Ratan 2010).

Evidence is not conclusive

- Predictions withing our framework depend on the magnitude of the loss aversion parameters.
- Stronger bidding in the Asymmetric Treatment $60 - 100$ requires a fairly high coefficient of loss aversion $(\lambda > 2)$.
- Our results are consistent with reference dependent preferences only for coefficients of loss aversion in the higher end of the commonly observed values

A new interpretation

Symmetry-Dependent Probability Distortion:

$$max_b \left(\frac{b}{B}\right)^{\frac{B}{v}} (v - b). \tag{2}$$

The conditional probability of winning gets distorted by the degree of asymmetry. When $v < B$ subjects feel they are disadvantaged and perceive a lower probability of winning even in the domain of profitable actions ($b \in [0, 60]$)

Optimal bid under Symmetry-Dependent Probability Distortion:

Optimal bid-to-value ratio $\frac{b^*}{v} = \frac{1}{1+v/B}$

Subjects bid the RNNE only when $v = B$.
When players feel they are disadvantaged they react overbidding.

Extension to the case of human bidders

Definition of symmetry

- The initial situation is defined by the space of valuations for both players.
- We define as symmetric an auction in which the private value of a bidder coincides with the median value of the known distribution of values of the opponent.
- Without loss of generality we impose a uniform distribution of $v \in [0;1]$ for both players, so that the median value of the opponent is simply 0.5.

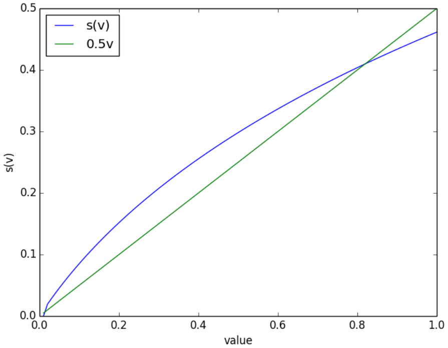
Objective function under Symmetry-Dependent Probability Distortion with human bidders:

$$b(v) \equiv \arg\max_{b} = (v - b)\{Pr[b(v_{-i}) \leq b]\}^{\frac{0.5}{v}}$$

Equilibrium bidding function $b(v) = ke^{\frac{0.5}{v}} - 0.5e^{\frac{0.5}{v}} EI(-\frac{0.5}{v})$

where EI is an exponential integral and k a constant.

Bids as function of private value



Nice properties

- Overbidding not limited to disadvantaged situations
- Concave Bidding function (often observed in the literature)
- Rationalizes underbidding of advantaged players (Elbittar 2009) as well as stronger overbidding of weaker subjects (Guth et al. 2005)

Conclusion

Summary:

- We study bidding in first-price sealed-bid auctions with independent private values (between subject experiment; one-shot decision against a single computerized opponent bidding uniformly)
- Average bids in the Symmetric treatments are very close to RNNE.
- We observe a much stronger overbidding when the value of the subject is lower than the upper end of the bidding space of the opponent.
- Evidence is inconsistent with objective functions which are linear in the probability of winning (expected utility, anticipated regret, and utility of winning)
- Non-linear probability weighting can account for the finding only for weighting function shapes lying outside of the commonly estimated range.
- Reference-dependent preferences would require very high degrees of loss aversion.
- We propose symmetry-dependent probability distortion as a new explanation of overbidding
- Under asymmetric likelihood of winning, disadvantaged players distort downward all the probabilities. As a consequence, they react overbidding.
- Such a symmetry-dependent probability distortion can be extended to account for stylized facts observed in auctions with multiple human bidders.